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Brief paper

Tracking and disturbance rejection for fully actuated mechanical systems☆

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ABSTRACT

In this paper, we solve the tracking and disturbance rejection problem for fully actuated passive mechanical systems. We assume that the reference signal r and its first two derivatives $\dot{r}$, $\ddot{r}$ are available to the controller and the disturbance signal d can be decomposed into a finite superposition of sine waves of arbitrary but known frequencies and an arbitrary L^2 signal. We combine the internal model principle with the ideas behind the *Slotine–Li* adaptive controller. The internal model-based adaptive controller that we propose causes the closed-loop state trajectories to be bounded, and the tracking error and its derivative to converge to zero, without any prior knowledge of the plant parameters. An important part of our results is that we prove the existence and uniqueness of the state trajectories of the closed-loop system.

1. Introduction

The internal model principle for LTI systems suggests that the dynamic structure of the exosystem must be included in the controller. For example, to eliminate the steady-state error for step reference or disturbance signals, we need integrators in the loop. If an internal model with transfer function $1/(s^2+\omega^2)$ (with suitable multiplicity) is in the feedback loop and the closed-loop system is stable, then we obtain tracking and/or disturbance rejection for sinusoidal reference and disturbance signals of frequency ω, see for example Davison and Goldenberg (1975). If the reference and disturbance signals are periodic, then the internal model principle leads to repetitive control (see for example Hara, Yamamoto, Omata, and Nakano (1988), and Weiss and Häfele (1999)).

The idea of an internal model has been generalized for output regulation of nonlinear systems by Byrnes, Delli Priscoli, and Isidori (1997) and Isidori (1995). In Byrnes et al. (1997) and Isidori (1995), the exogenous signal is generated by an exosystem and the existence of the controller requires the solvability of the Byrnes–Isidori regulator equations. Recent results on the output regulation of nonlinear systems can be found in Byrnes and Isidori (2003), Delli Priscoli (2004), Huang and Chen (2004) and Serrani, Isidori, and Marconi (2001).

In Jayawardhana and Weiss (2005, in press), a simple LTI internal model is used to solve the disturbance rejection problem for passive nonlinear plants. In Jayawardhana and Weiss (in press), the disturbance d is assumed to be of the form $d = d_0 + d_E$, where $d_0 \in L^2([0,\infty),\mathbb{R}^m)$, and d_E is generated by an LTI exosystem (as in (19)). No precise knowledge of the plant parameters is required in Jayawardhana and Weiss (in press). In this paper, the plant is a fully actuated mechanical system with the vector of generalized coordinates denoted by q, which should track a $\mathcal{C}^2$ reference signal r. We combine an LTI controller as in Jayawardhana and Weiss (in press) with a *Slotine–Li* type adaptive controller (see Slotine and Li (1988)) for rejecting a disturbance signal $d = d_0 + d_E$ as in Jayawardhana and Weiss (in press) and for asymptotically tracking r. We assume that the signals r, $\dot{r}$ and $\ddot{r}$ are available to the controller, but the controller does not know the parameters of the plant.

Our construction can be modified to allow the same LTI compensator to be combined with other passivity-based tracking controllers, for example, the passivity-based adaptive tracking controller in Slotine and Li (1989) or the adaptive tracking controller with adaptive friction compensator in Panteley, Ortega, and Gäfvert (1998).

In Scherpen and Ortega (1997), it is shown that by using the *Slotine–Li* controller and by adding to it a high gain proportional block from the tracking error to the input, the L^2 gain from the disturbance to the tracking error can be made arbitrarily small. However, this approach does not assure that the error converges to zero for a disturbance which is not in L^2. For a recent survey on tracking controllers for fully actuated mechanical systems we refer to Sage, de Mathelin, and Ostertag (1999). Results related to those

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in our paper have appeared in Bonivento, Gentili, and Paoli (2004). The controller in Bonivento et al. (2004) uses an adaptive internal model to find the frequencies of the disturbance, with the plant assumed to be known, while we use an adaptive controller to deal with uncertainty in the plant parameters (with the frequencies known).

We believe that our main contribution is to combine an internal model, which is usually considered for time-invariant systems, with the *Slotine–Li* controller, even though the latter leads to a time-varying system. Moreover, we allow an L^2 component in the disturbance signal, which is a new feature, and we are careful to prove the existence and uniqueness of state trajectories for the closed-loop system. We also show that both the tracking error and its time derivative tend to zero.

Our main results are stated and proved in Section 3. Due to the space constraints, we have no space to include simulation results. For this and for a design procedure for the internal model we refer to Jayawardhana (2006).

Notation. Throughout this paper, the inner product on any Hilbert space is denoted by $\langle\cdot,\cdot\rangle$ and $\mathbb{R}_+ = [0,\infty)$. We refer to Khalil (2000) and van der Schaft (2000) for basic concepts on nonlinear systems and on passivity theory. For a finite-dimensional vector x, we use the norm $\|x\| = \left(\sum_n |x_n|^2\right)^{\frac{1}{2}}$ and for matrices, we use the operator norm induced by $\|\cdot\|$ (the largest singular value). For a square matrix A, $\sigma(A)$ denotes the set of its eigenvalues. For any finite-dimensional vector space $\mathcal{V}$ endowed with a norm $\|\cdot\|_{\mathcal{V}}$, the space $L^2(\mathbb{R}_+,\mathcal{V})$ consists of all the measurable functions $f : \mathbb{R}_+ \to \mathcal{V}$ such that $\int_0^\infty \|f(t)\|_{\mathcal{V}}^2 dt < \infty$. The square-root of the last integral is denoted by $\|f\|_{L^2}$. For $f \in L^2(\mathbb{R}_+,\mathcal{V})$ and $T > 0$, we denote by f_T the truncation of f to $[0,T]$. The space $\mathcal{C}^1(\mathbb{R}^l,\mathbb{R}^p)$ consists of all the continuously differentiable functions $f:\mathbb{R}^l\to\mathbb{R}^p$, while $\mathcal{C}^2(\mathbb{R}_+)$ consists of all the twice continuously differentiable functions $r:\mathbb{R}_+\to\mathbb{R}$.

2. The Slotine–Li controller

Consider the problem of tracking a $\mathcal{C}^2$ reference signal r with the generalized coordinates q of a fully actuated mechanical system, without precise knowledge of the plant parameters. It is known that in the absence of disturbances, the *Slotine–Li* adaptive controller from Slotine and Li (1988) achieves asymptotic tracking of r with bounded state trajectories. In this section, first we show that the *Slotine–Li* feedback law applied to a fully actuated mechanical system produces a time-varying passive system. Using this, we generalize the results of Slotine and Li (1988) by allowing an L^2 disturbance to act on the plant. We show that, in spite of this disturbance, not only does the tracking error e tend to zero (as shown in van der Schaft (2000)) but also its time derivative $\dot e$.

We consider a plant **P** described by

$$\mathcal{M}(q)\ddot q + \mathcal{D}(q,\dot q)\dot q + g(q) = u, \tag{1}$$

which we call a *fully actuated mechanical system*. Such systems often originate from Euler–Lagrange equations for mechanical systems and they have been extensively studied, see Astolfi, Limebeer, Melchiorri, Tornambe, and Vinter (1997) and Ortega, Loría, Nicklasson, and Sira-Ramírez (1998). Here, $q(t)\in\mathbb{R}^n$ is the vector of *generalized coordinates*, $\mathcal{M}(q)$ is self-adjoint and

$$m_1 I \le \mathcal{M}(q) \le m_2 I, \quad \text{where } m_1, m_2 > 0, \tag{2}$$

$g(q)$ is a locally Lipschitz continuous function (which usually represents forces due to the potential energy) and $u(t)\in\mathbb{R}^n$ is the input (usually, forces or torques). The function $\mathcal{M}(\cdot)$ is assumed to be continuously differentiable and $\mathcal{D}(\cdot,\cdot)$ is assumed to be locally Lipschitz continuous. As usual, we denote $\dot{\mathcal{M}}(q,\dot q) = \sum_{j=1}^n \frac{\partial \mathcal{M}}{\partial q_j}\dot q_j$. The state of this system is the vector $\begin{bmatrix} q \\ \dot q\end{bmatrix}$. We assume that $J(q,\dot q) = \dot{\mathcal{M}}(q,\dot q) - 2\mathcal{D}(q,\dot q)$ satisfies $J^{\mathrm{T}}(q,\dot q) + J(q,\dot q) \le 0$, so that

$$\left\langle \left(\frac{1}{2}\dot{\mathcal{M}} - \mathcal{D}\right)a, a\right\rangle \le 0 \quad \forall a\in\mathbb{R}^n. \tag{3}$$

We remark that if $g(q) = (\nabla V(q))^{\mathrm{T}}$, where $V \in \mathcal{C}^1(\mathbb{R}^n,\mathbb{R}_+)$ is called the *potential energy*, then the plant **P** with output signal $\dot q$ is *passive* with respect to the storage function $H(q,\dot q) = \frac{1}{2}\langle \mathcal{M}(q)\dot q,\dot q\rangle + V(q)$, i.e., if a state trajectory exists then $\dot H \le \langle \dot q, u\rangle$. We mention that if $J^{\mathrm{T}} + J = 0$ then this system is *energy preserving*, meaning that $\dot H = \langle \dot q, u\rangle$.

We assume that $r \in \mathcal{C}^2(\mathbb{R}_+,\mathbb{R}^n)$ and the signals $r,\dot r,\ddot r$ are available to the controller. The input signal u is the sum of a disturbance signal d and the control input s (generated by the controller that we shall design), see Fig. 2(a). We assume that $\mathcal{M}$, $\mathcal{D}$ and g are not known exactly, but we can express them in terms of unknown real parameters $\theta_1,\theta_2,\ldots,\theta_m$ as follows:

$$\left.\begin{aligned}
\mathcal{M}(q) &= \sum_{i=1}^m \mathcal{M}_i(q)\theta_i + \mathcal{M}_0(q),\\
\mathcal{D}(q,\dot q) &= \sum_{i=1}^m \mathcal{D}_i(q,\dot q)\theta_i + \mathcal{D}_0(q,\dot q),\\
g(q) &= \sum_{i=1}^m g_i(q)\theta_i + g_0(q),
\end{aligned}\right\} \tag{4}$$

where $\mathcal{M}_i$ is of class $\mathcal{C}^1$ and $\mathcal{D}_i, g_i$ are locally Lipschitz continuous. For any $q,q_1,a,b\in\mathbb{R}^n$, we introduce the matrix $\Phi(q,q_1,a,b)\in\mathbb{R}^{n\times m}$ such that

$$\begin{aligned}\Phi(q,q_1,a,b)\theta &= \left(\sum_{i=1}^m \mathcal{M}_i(q)\theta_i\right)a + \left(\sum_{i=1}^m \mathcal{D}_i(q,q_1)\theta_i\right)b\\
&\quad + \sum_{i=1}^m g_i(q)\theta_i,\end{aligned} \tag{5}$$

where $\theta = \begin{bmatrix}\theta_1 & \theta_2 & \cdots & \theta_m\end{bmatrix}^{\mathrm{T}}$ is the parameter vector.

We describe a first feedback loop which is based on the *Slotine–Li* controller and which eliminates r from the picture, so that the problem is reduced to the input disturbance rejection problem. We denote by $\hat{\mathcal{M}}(q)$, $\hat{\mathcal{D}}(q,\dot q)$ and $\hat g(q)$ the estimates of $\mathcal{M}(q)$, $\mathcal{D}(q,\dot q)$ and $g(q)$ corresponding to the estimate $\hat\theta$ of the unknown parameter vector θ. (This means that $\hat{\mathcal{M}}(q)$ is obtained from (4) by replacing θ with $\hat\theta$, and similarly for $\hat{\mathcal{D}}(q,\dot q)$ and $\hat g(q)$.) Consider the feedback law

$$u = \hat{\mathcal{M}}\dot\xi + \hat{\mathcal{D}}\xi + \hat g + v, \tag{6}$$

where

$$\xi := \dot r + \Lambda(r-q), \quad \Lambda = \Lambda^{\mathrm{T}} \ge \mu I > 0, \tag{7}$$

and v is the new input signal, containing d and any other components of the control input z (to be designed). The estimated parameters $\hat\theta$ evolve according to

$$\dot{\hat\theta} = -\lambda\Phi(q,\dot q,\dot\xi,\xi)^{\mathrm{T}}\zeta, \tag{8}$$

where $\zeta = \dot q - \xi$ and $\lambda\in\mathbb{R}^{m\times m}$, $\lambda = \lambda^{\mathrm{T}} > 0$, see Fig. 1. Substituting (6) into (1) gives

$$\begin{aligned}\mathcal{M}(q)\dot\zeta + \mathcal{D}(q,\dot q)\zeta &= \left[\hat{\mathcal{M}}(q) - \mathcal{M}(q)\right]\dot\xi\\
&\quad + \left[\hat{\mathcal{D}}(q,\dot q) - \mathcal{D}(q,\dot q)\right]\xi + \hat g(q) - g(q) + v.\end{aligned} \tag{9}$$

Introducing the estimation error $\tilde\theta = \hat\theta - \theta$, we have $\hat{\mathcal{M}}(q) - \mathcal{M}(q) = \sum_{i=1}^m \mathcal{M}_i(q)\tilde\theta$, and we have similar formulas for $\hat{\mathcal{D}}(q,\dot q) - \mathcal{D}(q,\dot q)$ and $\hat g(q) - g(q)$. Now using (5), the formula (9) becomes

$$\mathcal{M}(q)\dot\zeta + \mathcal{D}(q,\dot q)\zeta = \Phi(q,\dot q,\dot\xi,\xi)\tilde\theta + v. \tag{10}$$

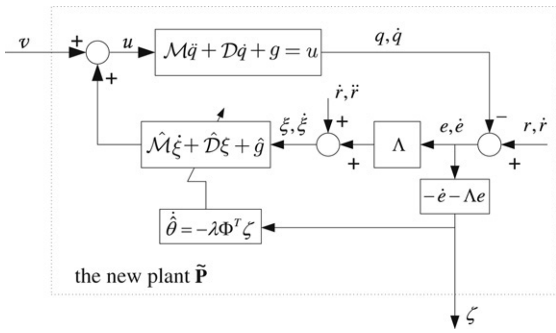

Fig. 1. The new plant $\tilde{\mathbf{P}}$ obtained after the feedback (6), in which the signal r is internally generated. The tracking error is e. This is a time-varying passive system with input v, state $(e, \zeta, \tilde{\theta})$, and output ζ.

From (8) it is clear that

$$\dot{\tilde{\theta}} = -\lambda \Phi(q, \dot{q}, \dot{\xi}, \xi)^{\mathrm{T}} \zeta. \tag{11}$$

A simple computation shows that, denoting $e = r - q$,

$$-\dot{e} - \Lambda e = \zeta. \tag{12}$$

The differential equations (10), (11) and (12) determine a new system $\tilde{\mathbf{P}}$ (shown in Fig. 1), for which it is natural to choose e, ζ and $\tilde{\theta}$ as state variables. What is disturbing in this system of equations is that (10), (11) depend also on $q, \dot{q}$. However, the state variables q and $\dot{q}$ of the plant can be expressed in terms of the state of $\tilde{\mathbf{P}}$: $q = r - e$, $\dot{q} = \dot{r} + \zeta + \Lambda e$ (remember that r and $\dot{r}$ are regarded as known functions). Thus, it is possible to rewrite (10) and (11) without using $q, \dot{q}$:

$$\mathcal{M}_r(e,t)\dot{\zeta} + \mathcal{D}_r(e,\zeta,t)\zeta = \Phi_r(e,\zeta,t)\tilde{\theta} + v, \tag{13}$$

$$\dot{\tilde{\theta}} = -\lambda \Phi_r(e,\zeta,t)^{\mathrm{T}} \zeta, \tag{14}$$

where, by definition,

$$\begin{aligned} \mathcal{M}_r(e,t) &= \mathcal{M}(r(t) - e), \\ \mathcal{D}_r(e,\zeta,t) &= \mathcal{D}(r(t) - e, \dot{r}(t) + \zeta + \Lambda e), \\ \Phi_r(e,\zeta,t) &= \Phi(r(t) - e, \dot{r}(t) + \zeta + \Lambda e, \\ &\quad \ddot{r}(t) - \Lambda(\Lambda e + \zeta), \dot{r}(t) + \Lambda e). \end{aligned} \tag{15}$$

Thus, a neat description of $\tilde{\mathbf{P}}$ consists of (12), (13) and (14). We derive a passivity property for $\tilde{\mathbf{P}}$. We denote $\dot{\mathcal{M}}_r = \frac{\partial \mathcal{M}_r}{\partial t} + \sum_{j=1}^n \frac{\partial \mathcal{M}_r}{\partial e_j} \dot{e}_j$, where $\frac{\partial \mathcal{M}_r(e)}{\partial t} = \sum_{j=1}^n \frac{\partial \mathcal{M}}{\partial q}(r - e)\dot{r}_j$. Note that $\dot{\mathcal{M}}_r(e,\zeta,t) = \dot{\mathcal{M}}(r(t) - e, \dot{r}(t) + \zeta + \Lambda e)$, so that (as in (3)),

$$\left\langle \left(\frac{1}{2}\dot{\mathcal{M}}_r - \mathcal{D}_r \right) a, a \right\rangle \leq 0, \quad \forall a \in \mathbb{R}^n. \tag{16}$$

Using

$$\tilde{H}(e,\zeta,\tilde{\theta},t) = \frac{1}{2}\langle \mathcal{M}_r(e,t)\zeta, \zeta \rangle + \frac{1}{2}\langle \tilde{\theta}, \lambda^{-1}\tilde{\theta} \rangle \tag{17}$$

as a storage function, $\tilde{\mathbf{P}}$ is a time-varying passive system with input v, state $(e, \zeta, \tilde{\theta})$ and output ζ. Indeed, using (13) and (16), we have

$$\begin{aligned} \dot{\tilde{H}} &= \langle \mathcal{M}_r \zeta, \dot{\zeta} \rangle + \frac{1}{2}\langle \dot{\mathcal{M}}_r \zeta, \zeta \rangle + \langle \tilde{\theta}, -\Phi_r^{\mathrm{T}} \zeta \rangle \\ &= -\langle \zeta, \mathcal{D}_r \zeta \rangle + \langle \zeta, \Phi_r \tilde{\theta} + v \rangle + \frac{1}{2}\langle \zeta, \dot{\mathcal{M}}_r \zeta \rangle - \langle \zeta, \Phi_r \tilde{\theta} \rangle \\ &\leq \langle \zeta, v \rangle. \end{aligned} \tag{18}$$

Assume that a disturbance d acts on the original system in (1), meaning that it is added to the input u. This has the same effect as adding d to the input v of the new system $\tilde{\mathbf{P}}$. We connect a stabilizing controller $\mathbf{C}$ to $\tilde{\mathbf{P}}$, as shown in Fig. 2(b). Thus, $v = y_c + d$ so that (according to (6)) $u = \hat{\mathcal{M}}\dot{\xi} + \hat{\mathcal{D}}\xi + \hat{g} + y_c + d$. Note that the total control input s from Fig. 2(a) is $s = \hat{\mathcal{M}}\dot{\xi} + \hat{\mathcal{D}}\xi + \hat{g} + y_c$.

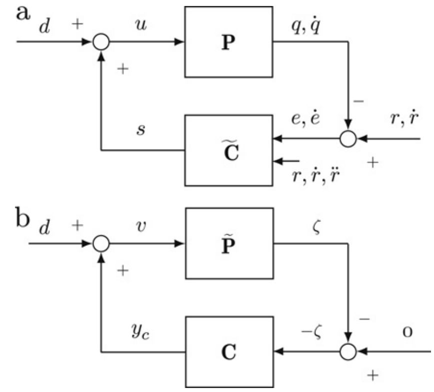

Fig. 2. The closed-loop system in (a) is as in Theorem 3.4. Note that the controller $\tilde{\mathbf{C}}$ needs r, $\dot{r}$ and $\ddot{r}$. This block diagram is equivalent to the one shown in (b), where $\tilde{\mathbf{P}}$ is the new plant from Fig. 1 and $\mathbf{C}$ is the stabilizing controller.

The well-known result from Slotine and Li (1988) and van der Schaft (2000) refers to the situation when $\mathbf{C}$ is a strictly positive constant matrix, $d \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and global solutions are assumed to exist. It states that in this situation, $e = r - q$ tends to zero. This is a particular case of our Proposition 3.3 in Section 4 (our conclusion is stronger even for this particular case, as we prove the existence of a unique global solution for every $d \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and also that $\dot{e}$ tends to zero).

3. The main result

In this section, we combine the *Slotine–Li* adaptive controller with a linear controller (an internal model) to achieve asymptotic tracking of r and at the same time the rejection of any input disturbance that can be decomposed into an L^2 component and a component generated by an exosystem.

Suppose now that the disturbance d applied to the control system in Fig. 2 can be decomposed as $d = d_0 + d_E$, where $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and d_E is generated by an exosystem $\mathbf{E}$. $\mathbf{E}$ is described by

$$\dot{w} = Sw, \qquad d_E(t) = C_w w(t), \tag{19}$$

where $C_w \in \mathbb{R}^{n \times p}$, $w(t) \in \mathbb{R}^p$ is the exosystem state, $S \in \mathbb{R}^{p \times p}$ has its eigenvalues on the imaginary axis and e^{St} is uniformly bounded for $t \geq 0$.

We shall need the following slight generalization of Barbălat's lemma.

Lemma 3.1. *Suppose that $\zeta \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ is uniformly continuous. Then $\lim_{t\to\infty} \zeta(t) = 0$.*

Proof. Since $\zeta \in L^2(\mathbb{R}_+, \mathbb{R}^n)$, it is a meagre function, as defined in Logemann and Ryan (2004). Now this lemma follows from Theorem 4.4 in the cited paper by taking there $x = \zeta$, $G = \mathbb{R}^n$ and $g(w) = \|w\|$. $\square$

Corollary 3.2. *Suppose that $\zeta \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and $\dot{\zeta} \in L^2(\mathbb{R}_+, \mathbb{R}^n) + L^\infty(\mathbb{R}_+, \mathbb{R}^n)$. Then $\lim_{t\to\infty} \zeta(t) = 0$.*

Indeed, any function ζ as in the corollary is uniformly continuous.

Before the next proposition, it will be convenient to summarize our assumptions. The plant $\mathbf{P}$ is described by (1), with assumptions on the functions $\mathcal{M}, \mathcal{D}$ and g as described there. We have a reference signal $r \in \mathcal{C}^2(\mathbb{R}_+, \mathbb{R}^n)$. The new plant $\tilde{\mathbf{P}}$ from Fig. 1 is obtained by connecting to $\mathbf{P}$ a *Slotine–Li* type controller, described by (5)–(8). We shall now propose another controller $\mathbf{C}$, to be connected to $\tilde{\mathbf{P}}$ as in Fig. 2(b), in order to reject the disturbance $d = d_0 + d_E$ described above. This controller is based on the internal model principle.

Proposition 3.3. *Suppose that $d = d_0 + d_E$, where $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and d_E is generated by the exosystem $\mathbf{E}$ from (19). Let the controller $\mathbf{C}$ with state $x_c(t) \in \mathbb{R}^l$, $l \geq p$, be given by*

$$\dot{x}_c = Ax_c - B\zeta, \qquad y_c = B^{\mathrm{T}}x_c - D\zeta, \tag{20}$$

where $A^{\mathrm{T}} + A = 0$, (B^{T}, A) is observable, $D = D^{\mathrm{T}} \geq kI$, $k > 0$ and there exists $\Sigma \in \mathbb{R}^{l\times p}$ such that

$$\Sigma S = A\Sigma, \qquad B^{\mathrm{T}}\Sigma + C_w = 0. \tag{21}$$

Then for every $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and for every $(e(0), \zeta(0), \tilde{\theta}(0), x_c(0), w(0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^l \times \mathbb{R}^p$, the state trajectory $(e, \zeta, \tilde{\theta}, x_c)$ of the closed-loop system $\mathbf{L}$ from Fig. 2(b) is uniquely defined for all $t \geq 0$, it is bounded and $\lim_{t\to\infty} e(t) = 0$.

Moreover, if $r, \dot{r}, \ddot{r}$ are bounded, then also $\lim_{t\to\infty} \zeta(t) = 0$ and $\lim_{t\to\infty} \dot{e}(t) = 0$.

Proof. Let us denote $\rho = x_c - \Sigma w$, then from (13), (19)–(21), (14) and (12) we have

$$\dot{e} = -\Lambda e - \zeta, \tag{22}$$

$$\begin{aligned}\mathcal{M}_r\dot{\zeta} &= -\mathcal{D}_r\zeta + \Phi_r\tilde{\theta} + B^{\mathrm{T}}(\rho + \Sigma w) + C_w w - D\zeta + d_0 \\ &= -\mathcal{D}_r\zeta + \Phi_r\tilde{\theta} + B^{\mathrm{T}}\rho - D\zeta + d_0,\end{aligned} \tag{23}$$

$$\dot{\tilde{\theta}} = -\lambda\Phi_r^{\mathrm{T}}\zeta, \tag{24}$$

$$\begin{aligned}\dot{\rho} &= A(\rho + \Sigma w) - B\zeta - \Sigma S w, \\ &= A\rho - B\zeta,\end{aligned} \tag{25}$$

where $(e, \zeta, \tilde{\theta}, \rho)$ is the state of the closed-loop system $\mathbf{L}$.

Using the notation $\mathcal{M}_r$ from (15), we introduce the storage function

$$H_{cl} = \frac{1}{2}\langle\mathcal{M}_r(e,t)\zeta, \zeta\rangle + \frac{\mu k}{2}\|e\|^2 + \frac{1}{2}\langle\tilde{\theta}, \lambda^{-1}\tilde{\theta}\rangle + \frac{1}{2}\|\rho\|^2$$

for the system $\mathbf{L}$. (Note that $H_{cl} = \tilde{H} + \frac{\mu k}{2}\|e\|^2 + \frac{1}{2}\|\rho\|^2$, where $\tilde{H}$ is defined in (17) and μ is as in (7).) Then using (22)–(24) and (16),

$$\begin{aligned}\dot{H}_{cl} &= \langle\zeta, -\mathcal{D}_r\zeta + \Phi_r\tilde{\theta} + B^{\mathrm{T}}\rho - D\zeta + d_0\rangle + \frac{1}{2}\langle\dot{\mathcal{M}}_r\zeta, \zeta\rangle \\ &\quad + \mu k\langle e, -\Lambda e - \zeta\rangle + \langle\tilde{\theta}, -\Phi_r^{\mathrm{T}}\zeta\rangle + \langle\rho, A\rho - B\zeta\rangle \\ &\leq \langle\zeta, d_0\rangle - \left\langle\begin{bmatrix}\zeta\\ e\end{bmatrix}, \begin{bmatrix}kI & \mu k/2I\\ \mu k/2I & \mu^2 kI\end{bmatrix}\begin{bmatrix}\zeta\\ e\end{bmatrix}\right\rangle \\ &\leq \langle\zeta, d_0\rangle - c_1\|\zeta\|^2,\end{aligned} \tag{26}$$

where $c_1 > 0$ satisfies $\begin{bmatrix}kI & \mu k/2I\\ \mu k/2I & \mu^2 kI\end{bmatrix} \geq c_1 I$.

It can be shown that the closed-loop system equations satisfy the Assumptions **(S1)**–**(S2)** in Appendix A. Indeed, the closed-loop system equation can be written as

$$\dot{\tilde{z}} = \tilde{F}(t, \tilde{z}, d_0) = \begin{bmatrix}f(t,z)\\ 0\end{bmatrix} + \begin{bmatrix}0\\ (\mathcal{M}_r(e,t))^{-1}(d_0 - D\zeta)\\ 0\\ A\rho - B\zeta\end{bmatrix},$$

where $\tilde{z} = \begin{bmatrix}e\\ \zeta\\ \tilde{\theta}\\ \rho\end{bmatrix}$ and

$$f(t,z) = \begin{bmatrix}-\Lambda e - \zeta\\ (\mathcal{M}_r)^{-1}(-\mathcal{D}_r\zeta + \Phi_r\tilde{\theta})\\ -\lambda\Phi_r^{\mathrm{T}}\zeta\end{bmatrix}. \tag{27}$$

Using Lemma B.2 in Appendix B and since $(\mathcal{M}_r(e,t))^{-1}$ is locally Lipschitz, it can be checked that for any compact set $\mathcal{B} \subset \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^l$ there exist locally bounded functions $\alpha, \gamma : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\|\tilde{F}(t, z_1, a) - \tilde{F}(t, z_2, a)\| \leq (\alpha(t) + \gamma(t)\|a\|)\,\|z_1 - z_2\|$$

holds for all $z_1, z_2 \in \mathcal{B}$, $a \in \mathbb{R}^n$ and for all $t \in \mathbb{R}_+$.

It is clear that $\tilde{F}$ is continuous and $(\mathcal{M}_r)^{-1}$ is bounded from above, hence for every $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$, there exists a locally bounded function $\beta : \mathbb{R}_+ \to \mathbb{R}_+$ and a constant $c_2 > 0$ such that $\|\tilde{F}(t,b,a)\| \leq \beta(t) + c_2\|a\|$ for all $t \in \mathbb{R}_+$. These imply that the function $\tilde{F}$ satisfies **(S1)**–**(S2)** (as defined in Appendix A). Since H_{cl} is proper, it follows from Proposition A.1 that for any $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and for any initial state $(e(0), \zeta(0), \tilde{\theta}(0), x_c(0), w(0))$, there exists a unique global solution $(e, \zeta, \tilde{\theta}, \rho)$ of the closed-loop system and the state trajectory $(e, \zeta, \tilde{\theta}, \rho)$ is bounded. Since ρ and w are bounded and $\rho = x_c - \Sigma w$, we see that x_c is also bounded.

Using (26), $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ implies that also $\zeta \in L^2(\mathbb{R}_+, \mathbb{R}^n)$, see Lemma 6.5 in Khalil (2000). Since the system (22) with input ζ and output e is stable and $\zeta \in L^2(\mathbb{R}_+, \mathbb{R}^n)$, it follows that $e \in L^2(\mathbb{R}_+, \mathbb{R}^n)$. Since $\dot{e} = -\Lambda e - \zeta$, we have $\dot{e} \in L^2(\mathbb{R}_+, \mathbb{R}^n)$. By Corollary 3.2 we get that $\lim_{t\to\infty} e(t) = 0$.

To prove the last part of the proposition, we assume that $r, \dot{r}, \ddot{r}$ are bounded. We rewrite (23) as follows:

$$\dot{\zeta} = \mathcal{M}_r^{-1}\left(-\mathcal{D}_r\zeta + \Phi_r\tilde{\theta} + B^{\mathrm{T}}\rho - D\zeta + d_0\right). \tag{28}$$

It follows from (2) that $\mathcal{M}_r(e,t)^{-1} \leq m_1^{-1}I$. Since $e, \zeta, \tilde{\theta}$ and $r, \dot{r}, \ddot{r}$ are bounded, the continuity of $\mathcal{D}$ and Φ implies that $\mathcal{D}_r(e(t), \zeta(t), t)$ and $\Phi_r(e(t), \zeta(t), t)$ are bounded functions of t. Since $\zeta, d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$, the function $G(t) = \mathcal{M}_r(e,t)^{-1}[-\mathcal{D}_r(e,\zeta,t)\zeta(t) + d_0(t) - D\zeta(t)]$ is in $L^2(\mathbb{R}_+, \mathbb{R}^n)$. Since $\tilde{\theta} \in L^\infty(\mathbb{R}_+, \mathbb{R}^n)$ and $\rho \in L^\infty(\mathbb{R}_+, \mathbb{R}^l)$, it implies that the function $H(t) = \mathcal{M}_r(e,t)^{-1}\left[\Phi_r(e,\zeta,t)\tilde{\theta}(t) + B^{\mathrm{T}}\rho(t)\right]$ is in $L^\infty(\mathbb{R}_+, \mathbb{R}^n)$. Thus, (28) shows that $\dot{\zeta} \in L^2(\mathbb{R}_+, \mathbb{R}^n) + L^\infty(\mathbb{R}_+, \mathbb{R}^n)$. By Corollary 3.2 we obtain $\lim_{t\to\infty} \zeta(t) = 0$. Now from (12) it follows that $\lim_{t\to\infty} \dot{e}(t) = 0$. □

The following theorem, which is our main result, is just a reformulation of Proposition 3.3 for bounded $r, \dot{r}, \ddot{r}$, in terms of the original plant $\mathbf{P}$.

Theorem 3.4. *Consider the system $\mathbf{P}$ as in (1) with outputs q and $\dot{q}$, the reference $r \in \mathcal{C}^2(\mathbb{R}_+, \mathbb{R}^n)$, where $r, \dot{r}, \ddot{r}$ are bounded and the disturbance $d = d_0 + d_E$, where $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$ and d_E is generated by the exosystem $\mathbf{E}$ from (19). The controller $\tilde{\mathbf{C}}$ has the state equations*

$$\dot{x}_c = Ax_c - B\zeta, \qquad \dot{\hat{\theta}} = -\lambda\Phi(q, \dot{q}, \dot{\xi}, \xi)^{\mathrm{T}}\zeta, \tag{29}$$

where $x_c(t) \in \mathbb{R}^l$, $l \geq p$, $e = r - q$, $\zeta(t) \in \mathbb{R}^n$ is defined in (12), $A \in \mathbb{R}^{l\times l}$, $A^{\mathrm{T}} + A = 0$, $B \in \mathbb{R}^{l\times n}$. Here, $\hat{\theta}(t) \in \mathbb{R}^m$, $\xi = \dot{r} + \Lambda e$, $\lambda \in \mathbb{R}^{m\times m}$, $\lambda^{\mathrm{T}} = \lambda > 0$ and Φ is as in (5). The controller generates the signal

$$s = \hat{\mathcal{M}}\dot{\xi} + \hat{\mathcal{D}}\xi + \hat{g} + B^{\mathrm{T}}x_c - D\zeta,$$

where $s(t) \in \mathbb{R}^n$ and $D = D^{\mathrm{T}} \geq kI$, $k > 0$. We assume that (B^{T}, A) is observable and there exists $\Sigma \in \mathbb{R}^{l\times p}$ which satisfies (21).

Then for every $(q(0), \dot{q}(0), \hat{\theta}(0), x_c(0), w(0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^l \times \mathbb{R}^p$ and for every $d_0 \in L^2(\mathbb{R}_+, \mathbb{R}^n)$, the state trajectory $(q, \dot{q}, \hat{\theta}, x_c)$ of the closed-loop system $\mathbf{L}$ shown in Fig. 2(a) is uniquely defined for all $t \geq 0$, it is bounded, $\lim_{t\to\infty} e(t) = 0$ and $\lim_{t\to\infty} \dot{e}(t) = 0$.

Proof. The closed-loop system $\mathbf{L}$ from Fig. 2(a) is the same as the one in Fig. 2(b). The state variables for Fig. 2(a) can be expressed in terms of those for Fig. 2(b) as follows: $q = r - e$, $\dot q = \dot r + \zeta + \Lambda e$, $\hat\theta = \theta + \tilde\theta$. Now all the claims follow from Proposition 3.3. □

We have seen in Theorem 3.4 that the internal model based compensator described by (29) solves the tracking and disturbance rejection problem for $\mathbf{P}$ if the equations (21) have a solution. However, this result by itself does not indicate any practical way to construct A and B, since C_w and S are not known (we only know the frequencies of the signal). We can construct A and B using only the eigenvalues of S as our input data (these correspond to the frequencies of the exosystems $\mathbf{E}$).

Let $\chi(s) = s^\kappa + a_{\kappa-1}s^{\kappa-1}\cdots + a_1 s + a_0$ be the minimal polynomial of $S \in \mathbb{R}^{p\times p}$, so that $\chi(S) = 0$, $a_j \ge 0$, $\kappa \le p$ and χ has only simple zeros, all on $\mathrm{i}\mathbb{R}$. Suppose that $S_{\min} \in \mathbb{R}^{\kappa\times\kappa}$ is such that $S_{\min} + S_{\min}^{\mathrm{T}} = 0$ and its characteristic polynomial is χ. If $0 \in \sigma(S)$ then we can use

$$S_{\min} = \operatorname{diag}\{0, \Omega_1, \Omega_2, \ldots, \Omega_\nu\}, \tag{30}$$

where for each $k = 1, \ldots, \nu$, $\Omega_k = \begin{bmatrix} 0 & -\omega_k \\ \omega_k & 0 \end{bmatrix}$ for some $\omega_k \in \mathbb{R}\setminus\{0\}$ and $\omega_k \ne \omega_j$ for $k \ne j$. The set $\sigma(S_{\min}) = \sigma(S)$ contains 0 and $\pm\mathrm{i}\omega_k (k = 1, \ldots, \nu)$ (0 and ω_k are the known frequencies of the disturbance signal). If $0 \notin \sigma(S)$, then we omit the first line and the first column in (30), so that $\sigma(S_{\min})$ contains only $\pm\mathrm{i}\omega_k$.

For $i = 1, \ldots, m$, let $\Gamma_i \in \mathbb{R}^{\kappa\times 1}$ be such that $(\Gamma_i^{\mathrm{T}}, S_{\min})$ is observable (the m vectors Γ_i may be taken equal). Using $S_{\min}$ and $\Gamma_i, i = 1, \ldots, m$, the matrices A and B that satisfy the conditions in Theorem 3.4 can be defined by

$$\left.\begin{aligned} A &= \operatorname{diag}\{S_{\min}, S_{\min}, \ldots, S_{\min}\}, \\ B &= \operatorname{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_n\}, \end{aligned}\right\} \tag{31}$$

see Jayawardhana (2006) for the proof. The controller output signal s from Fig. 2(a) will be

$$s = \hat{\mathcal{M}}\dot\xi + \hat{\mathcal{D}}\xi + \hat g + B^{\mathrm{T}}x_c - D\zeta,$$

where $D = D^{\mathrm{T}} \ge kI$, $k > 0$.

4. Conclusions

We have shown that a simple LTI internal model-based controller can be combined with the *Slotine–Li* controller to solve the tracking and disturbance rejection problem for a class of fully actuated passive mechanical systems. The parameter estimation errors will be bounded (they do not have to converge to zero) but we prove that the tracking error and its derivative tend to zero.

Appendix A. Existence of global solutions for time-varying passive systems

Consider the time-varying passive systems $\mathbf{P}$ given by the state equations

$$\dot x = f(t, x, u) \qquad y = h(t, x, u), \tag{A.1}$$

where $x(t) \in \mathbb{R}^n$, $u(t), y(t) \in \mathbb{R}^m$, the functions $f \in \mathcal{C}(\mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^n)$ and $h \in \mathcal{C}(\mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^m)$. We assume that there exists $H \in \mathcal{C}^1(\mathbb{R}_+ \times \mathbb{R}^n, \mathbb{R}_+)$ such that $H(t, x)$ satisfies

$$\dot H = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial x} f(t, x, u) \le \langle y, u\rangle - k\|y\|^2, \tag{A.2}$$

where $k > 0$. Under these assumptions, we say that $\mathbf{P}$ is a *time-varying strictly output passive* system with *time-varying storage function* H.

The time-varying storage function H is called *proper* if for every $c > 0$ and for every $t \ge 0$ the set $\{x \in \mathbb{R}^n \mid H(t, x) \le c\}$ is compact.

We cannot know if this system of differential equations has a global solution for every initial state $x(0)$ and every input function $u \in L^2(\mathbb{R}_+, \mathbb{R}^n)$, and even if it does, we do not know if the solution is unique.

We impose the following conditions on f to ensure the existence and uniqueness of global solutions:

(S1) For every compact set $\mathcal{B} \subset \mathbb{R}^n$, there exists a locally bounded function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ and a locally integrable function $\tau : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\|f(t, x_1, u) - f(t, x_2, u)\| \le \left[\gamma(t)\|u\|^2 + \tau(t)\right]\|x_1 - x_2\|,$$

for all $u \in \mathbb{R}^m$, $x_1, x_2 \in \mathcal{B}$ and for almost every $t \in \mathbb{R}_+$.

(S2) For each fixed $a \in \mathbb{R}^n$, there exists a constant $c > 0$ and a locally integrable function $\upsilon : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\|f(t, a, u)\| \le c\|u\|^2 + \upsilon(t)$, for all $u \in \mathbb{R}^m$ and for almost every $t \in \mathbb{R}_+$.

Proposition A.1. *Let the plant* $\mathbf{P}$ *defined by* (A.1) *satisfy* **(S1)–(S2)**. *Assume that* $\mathbf{P}$ *has a proper time-varying storage function* H *and* (A.2) *holds with* $k > 0$.
Then for every initial state $x(0) \in \mathbb{R}^n$ *and for every* $u \in L^2(\mathbb{R}_+, \mathbb{R}^m)$, *the state trajectory* x *of* $\mathbf{P}$ *is defined uniquely for all* $t \ge 0$ *and it is bounded.*

The proof of this proposition can be obtained by applying the theorem on the existence and uniqueness of local solutions for systems of the form (A.1) (for example, Theorem 36 in Sontag (1990)), and then using the passivity property to show that the solutions are bounded, and hence their interval of existence in infinite. The details can be found in Jayawardhana (2006).

In Jayawardhana (2006) and in Jayawardhana and Weiss (in press), an example of a strictly output passive system which does not satisfy **(S1)–(S2)** is given and it is shown that for some input $u \in L^2$ it does not have a solution on any interval $[0, \delta)$, $\delta > 0$.

Appendix B. Conditions (S2) for a fully-actuated mechanical system with a *Slotine–Li* controller

We shall show that the conditions **(S2)** in Appendix A are satisfied for the time-varying system $\tilde{\mathbf{P}}$ obtained from the fully actuated mechanical system (1) with the *Slotine–Li* controller.

Lemma B.1. *Suppose that* $f_1 \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^{p_1\times p_2})$ *and* $f_2 \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^{p_2\times p_3})$ *are locally Lipschitz. Then the function* $f : x \mapsto f_1(x)f_2(x)$ *is locally Lipschitz.*

The proof of this is easy and left to the reader.

Let us recall the state equations of $\tilde{\mathbf{P}}$ from Section 2 and the assumptions stated there:

$$\dot z = f(t, z) + \begin{bmatrix} 0 \\ (\mathcal{M}_r(e, t))^{-1}\upsilon \\ 0 \end{bmatrix}, \tag{B.1}$$

where $z(t) = \begin{bmatrix} e(t) \\ \zeta(t) \\ \tilde\theta(t) \end{bmatrix} \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$ and f is given by (27). Remember that $\mathcal{M}_r$, $\mathcal{D}_r$ and Φ_r are defined in terms of $\mathcal{M}$, $\mathcal{D}$ and g, see (15), where $\mathcal{M} \in \mathcal{C}^1$ and $\mathcal{D}$, g are locally Lipschitz. The signal r is assumed to be in $\mathcal{C}^2$. It follows that $\mathcal{M}_r \in \mathcal{C}^1(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}^n)$ and $\mathcal{D}_r, \Phi_r \in \mathcal{C}(\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}^n)$. We have $m_1 I \le \mathcal{M}_r(e, t) \le m_2 I$ for all $(e, t) \in \mathbb{R}^n \times \mathbb{R}_+$ where $m_1, m_2 > 0$, according to (2). It follows that $\frac{1}{m_2}I \le (\mathcal{M}_r(e, t))^{-1} \le \frac{1}{m_1}I$ for all $(e, t) \in \mathbb{R}^n \times \mathbb{R}$ and $(\mathcal{M}_r)^{-1} \in \mathcal{C}^1(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}^n)$.

Lemma B.2. *For any compact set* $\mathcal{B} \subset \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$, *there exists a locally bounded function* $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ *such that* $\|f(t, z_1) - f(t, z_2)\| \le \alpha(t)\|z_1 - z_2\|$ *holds for all* $z_1, z_2 \in \mathcal{B}$ *and for all* $t \in \mathbb{R}_+$.

Proof. Since $r \in \mathcal{C}^2(\mathbb{R}_+, \mathbb{R}^n)$, the signals $r, \dot r$ and $\ddot r$ are locally bounded and continuous. Take any compact set $\mathcal{B} \subset \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$.

Using the local Lipschitz assumption on $\mathcal{D}$ and using Lemma B.1, there exists a locally bounded function $\gamma_1 : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\begin{aligned}
&\|\mathcal{D}_r(e_1, \zeta_1, t)\zeta_1 - \mathcal{D}_r(e_2, \zeta_2, t)\zeta_2\| \\
&\quad = \|\mathcal{D}(r(t) - e_1, \dot{r}(t) + \zeta_1 + \Lambda e_1)\zeta_1 \\
&\qquad - \mathcal{D}(r(t) - e_2, \dot{r}(t) + \zeta_2 + \Lambda e_2)\zeta_2\| \\
&\quad \le \gamma_1(t) \left\| \begin{bmatrix} e_1 - e_2 \\ \zeta_1 - \zeta_2 \end{bmatrix} \right\| \quad t \in \mathbb{R}_+
\end{aligned} \tag{B.2}$$

holds for all $z_1 = \begin{bmatrix} e_1 \\ \zeta_1 \\ \tilde{\theta}_1 \end{bmatrix}, z_2 = \begin{bmatrix} e_2 \\ \zeta_2 \\ \tilde{\theta}_2 \end{bmatrix}$ in $\mathcal{B}$.

Denote $\Phi_i(q, \dot{q}, a, b) = \mathcal{M}_i(q)a + \mathcal{D}_i(q, \dot{q})b + g_i(q)$, $i = 1, 2, \ldots, m$ where $\mathcal{M}_i$, $\mathcal{D}_i$ and g_i are as in (4). Using the local Lipschitz assumption on $\mathcal{M}_i$, $\mathcal{D}_i$ and g_i and using Lemma B.1 we conclude that Φ_i is locally Lipschitz.

As in (5), denote $\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_m \end{bmatrix}$. Since Φ_i is locally Lipschitz for all $i = 1, 2 \ldots m$ and $r, \dot{r}, \ddot{r}$ are locally bounded, it can be checked that there exists a locally bounded function $\gamma_2 : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\|\Phi_r(e_1, \zeta_1, t)\tilde{\theta}_1 - \Phi_r(e_2, \zeta_2, t)\tilde{\theta}_2\| \le \gamma_2(t) \|z_1 - z_2\| \tag{B.3}$$

holds for all $z_1, z_2 \in \mathcal{B}$ and for all $t \in \mathbb{R}_+$. This condition is also satisfied for $\Phi_r(e, \zeta, t)^{\mathrm{T}}\zeta$ with a locally bounded function $\gamma_3 : \mathbb{R}_+ \to \mathbb{R}_+$.

Denote

$$\Psi(z, t) = \mathcal{M}_r^{-1}(e, t)\left(-\mathcal{D}_r(e, \zeta, t)\zeta + \Phi_r(e, \zeta, t)\tilde{\theta}\right).$$

Note that $(\mathcal{M}_r(e, t))^{-1}$ is locally Lipschitz w.r.t. e, with the Lipschitz constant depending on $r(t)$. Using this fact, (B.2), (B.3) and Lemma B.1, it follows that there exists a locally bounded function $\gamma_4 : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\|\Psi(e_1, \zeta_1, \tilde{\theta}_1, t) - \Psi(e_2, \zeta_2, \tilde{\theta}_2, t)\| \le \gamma_4(t) \|z_1 - z_2\|$$

holds for all $z_1, z_2 \in \mathcal{B}$ and for all $t \in \mathbb{R}_+$.

It is easy to see that there exists a positive constant $c_2 > 0$ such that

$$\|(-\Lambda e_1 - \zeta_1) - (-\Lambda e_2 - \zeta_2)\| \le c_2 \left\| \begin{bmatrix} e_1 - e_2 \\ \zeta_1 - \zeta_2 \end{bmatrix} \right\|$$

holds for all $z_1, z_2 \in \mathcal{B}$.

The lemma is proved with $\alpha(t) = \lambda\gamma_3(t) + \gamma_4(t) + c_2$. □

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